Real-Time Implementation of Nonlinear Predictive Control

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Outline

- Limitations of linear model predictive control
- Introduction to nonlinear model predictive control
- Real-time implementation issues
- Nonlinear control of an air separation column
- Final comments

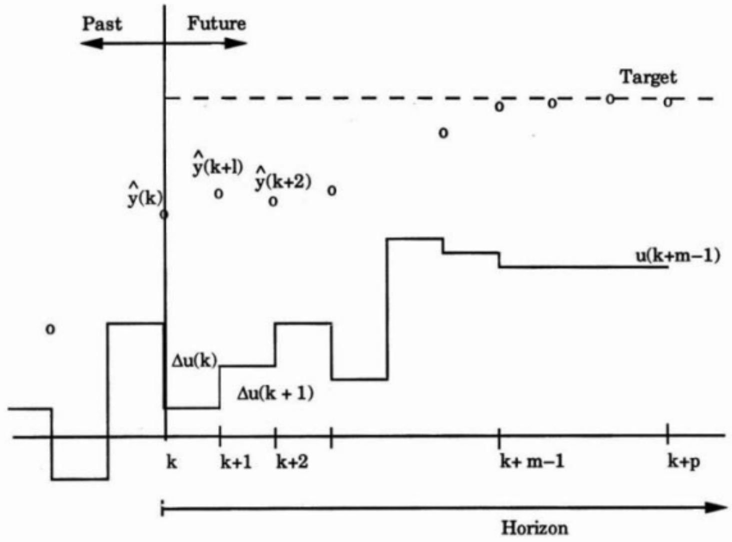


Figure 27.3. The elements of DMC: The "reference trajectory" is the set-point line.

1. Limitations of Linear Model Predictive Control

Linear Model Predictive Control (LMPC)

- Background
  - Constrained multivariable control technology
  - Requires availability of linear dynamic model
  - Chemical process industry standard
- Real-time implementation
  - Repeated on-line solution of optimization problem
  - Receding horizon formulation
  - Computationally efficient & robust quadratic program
- Commercial technology
  - DMCplus (Aspen Technology)
  - RMPCT (Honeywell)
  - Many others

Dynamic Matrix Control (DMC)

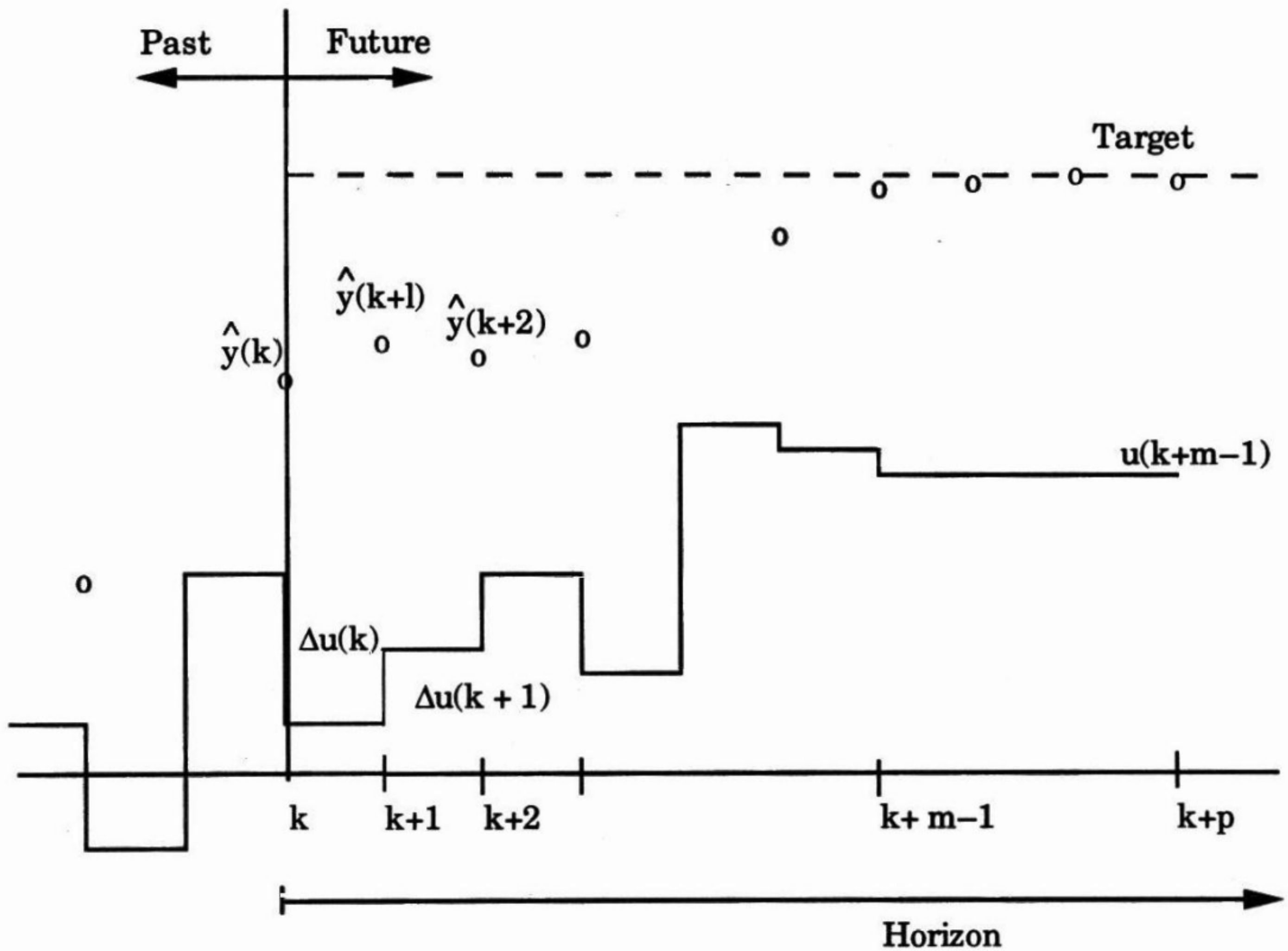


Figure 27.3. The elements of DMC: The "reference trajectory" is the set-point line.

Standard LMPC Formulation

$$\underset{\Delta U_{f,k}}{\textbf{Min}}\ \Phi = \left[Y_{k+1} - Y^{\text{sp}}\right]^T W_e \left[Y_{k+1} - Y^{\text{sp}}\right] + \Delta U^T_{f,k} W_{\Delta U} \Delta U_{f,k}$$

$$+ \left[U_{f,k} - U^{\text{sp}}\right]^T W_e \left[U_{f,k} - U^{\text{sp}}\right]$$

$$\textbf{S.t} \quad Y_{k+1} = S_f \Delta U_{f,k} + \left[S_p \Delta U_{p,k} + S_N u_{k-N+1} + d_k\right]$$

$$U_{f,k} = u_{k-1} + D\Delta U_{f,k}$$

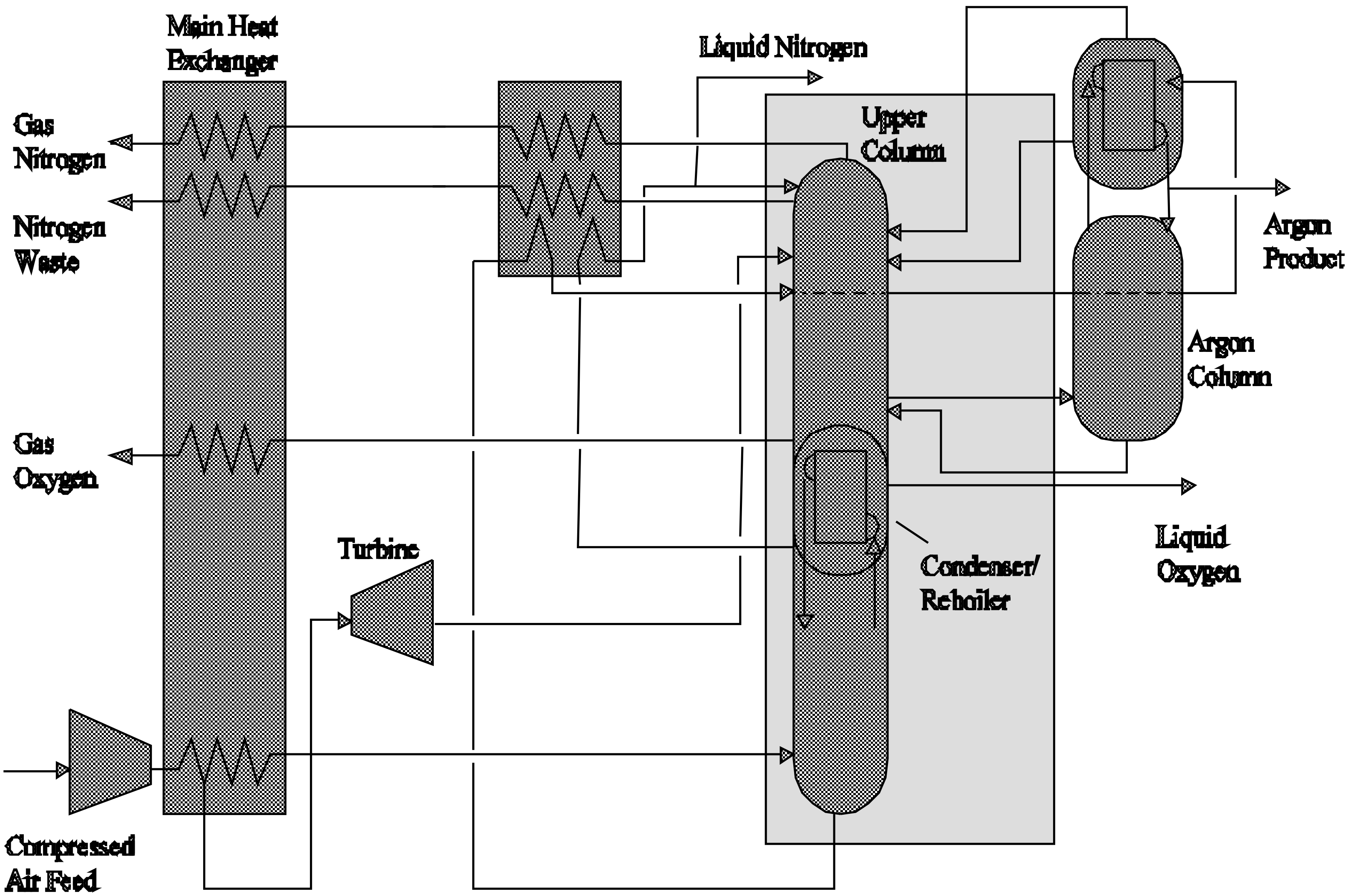
$$U_{\min} \le U_{f,k} \le U_{\max}$$

$$\Delta U_{\min} \le \Delta U_{f,k} \le \Delta U_{\max}$$

$$\mathrm{Y}_{\min} \le Y_{k+1} \le Y_{\max}$$

- Controlled variables (*Y*)
  - Penalize deviations from target values (Y^{sp})
  - Hard lower & upper bound constraints
- Manipulated variables (*U*)
  - Manipulated to minimize objective function
  - Penalize deviations for target values (U^{sp})
  - Hard constraints on absolute values & rate-of-changes
- Equality constraints
  - Linear step response model identified from plant tests
- Tuning parameters
  - Sampling time
  - Prediction horizon
  - Control horizon
  - Weighting matrices

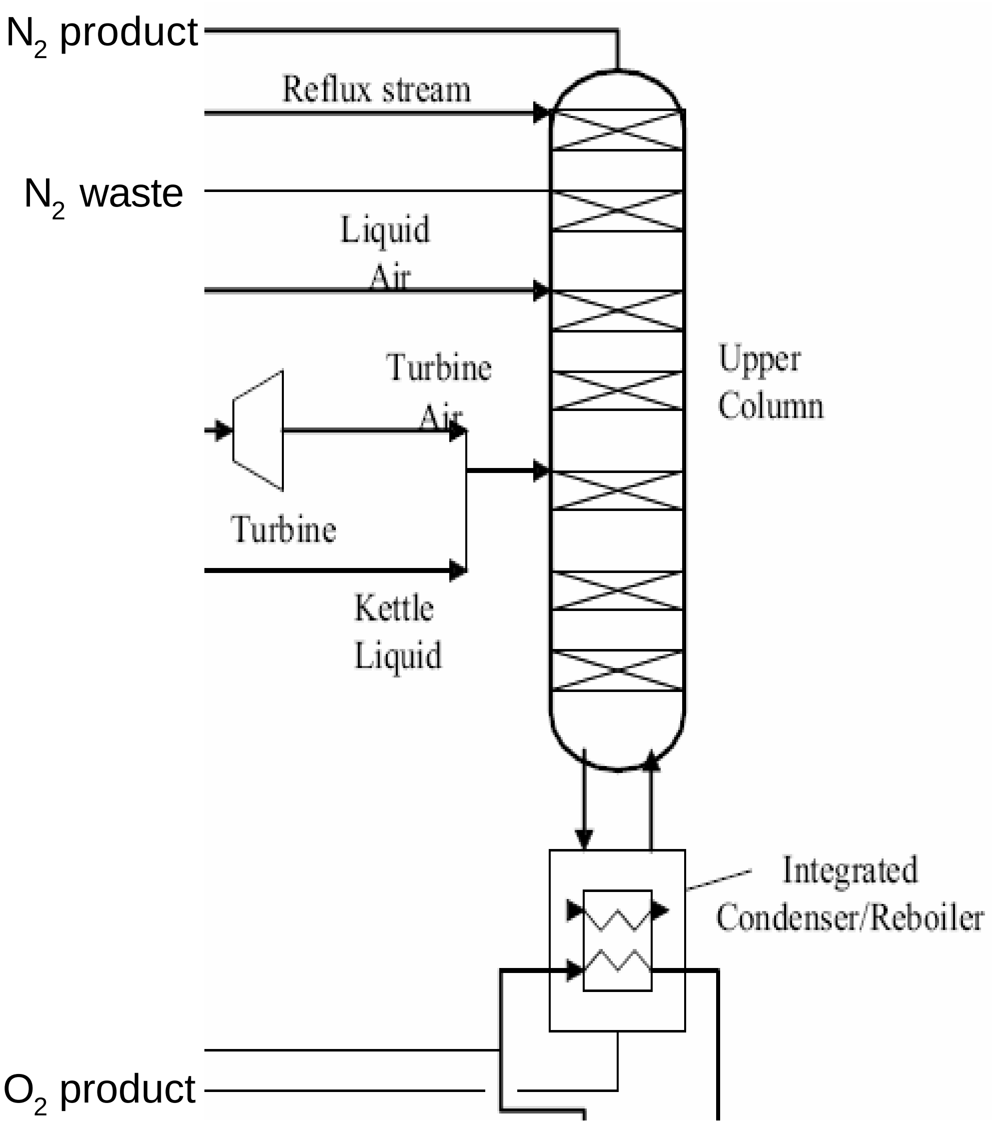
Triple Column Air Separation Plant



Air Separation Plant Control

- Current practice
  - Empirical linear models & linear model predictive control
  - Adequate for small, well defined operating regimes
- Production rate changes
  - Motivated by electricity industry deregulation
  - Exaggerated nonlinearities
- Plant startup & shutdown
  - Operation over large operating regimes
  - Strong nonlinearities
- Future needs
  - More dynamic operating philosophy
  - Nonlinear behavior more pronounced

Upper Column



- Packed column modeled with equilibrium stages
- Multiple liquid distributors
- Feeds
  - Reflux from lower column
  - Liquid air
  - Turbine air
- Withdrawals
  - N_2 product
  - N_2 waste
  - O_2 product

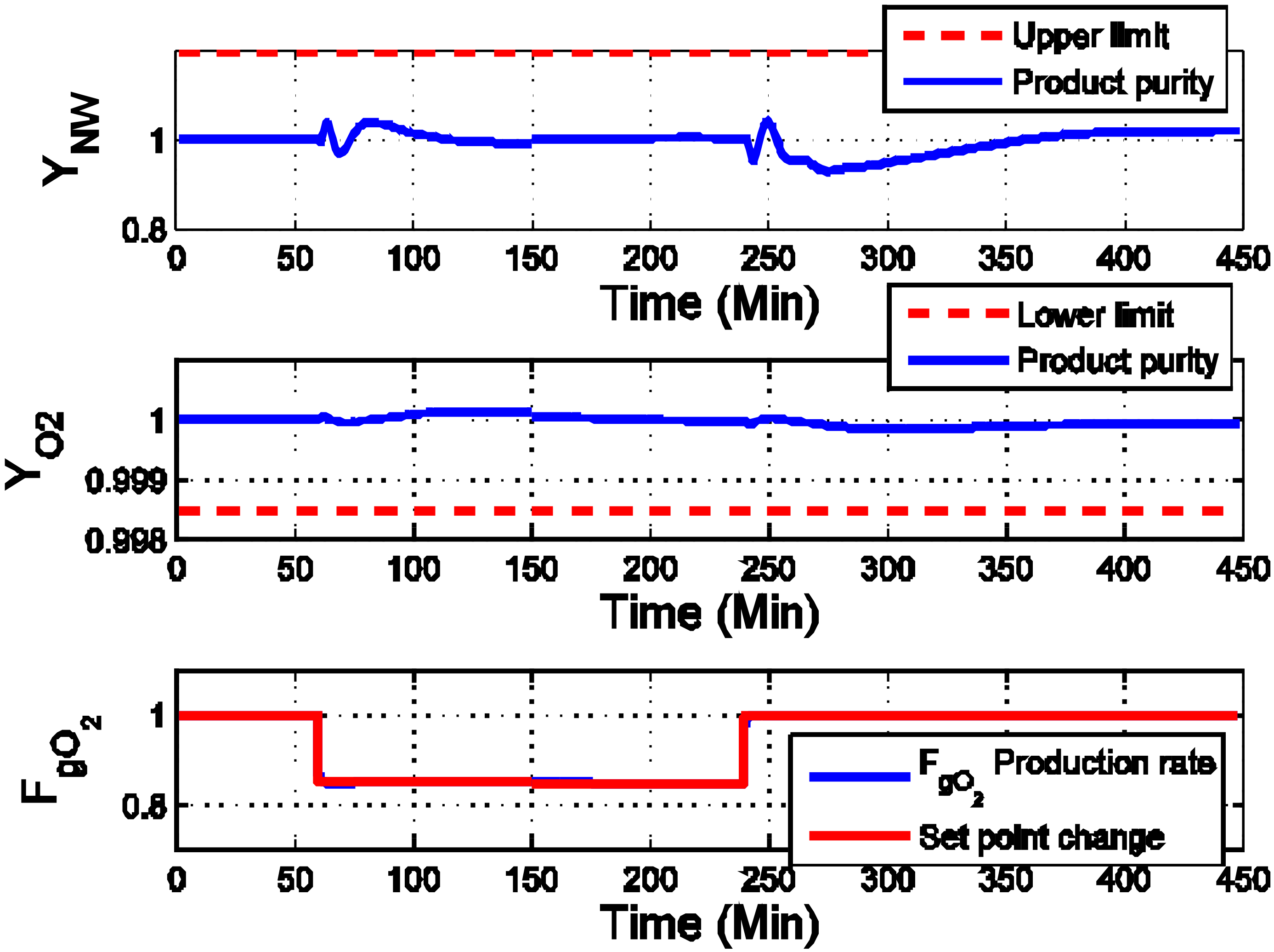
Aspen Simulation Model

- Column model RadFrac
  - Dynamic component balances
  - Steady-state energy balances
- Non-ideal vapor-liquid equilibrium
  - NRTL for liquid phase
  - Peng-Robinson for vapor phase
  - Thermodynamic property data provided by Praxair
- PID controllers
  - Reboiler level & overhead pressure
- Coupling to lower column
  - Lower column effect on upper column described by empirical linear models identified from an Aspen model

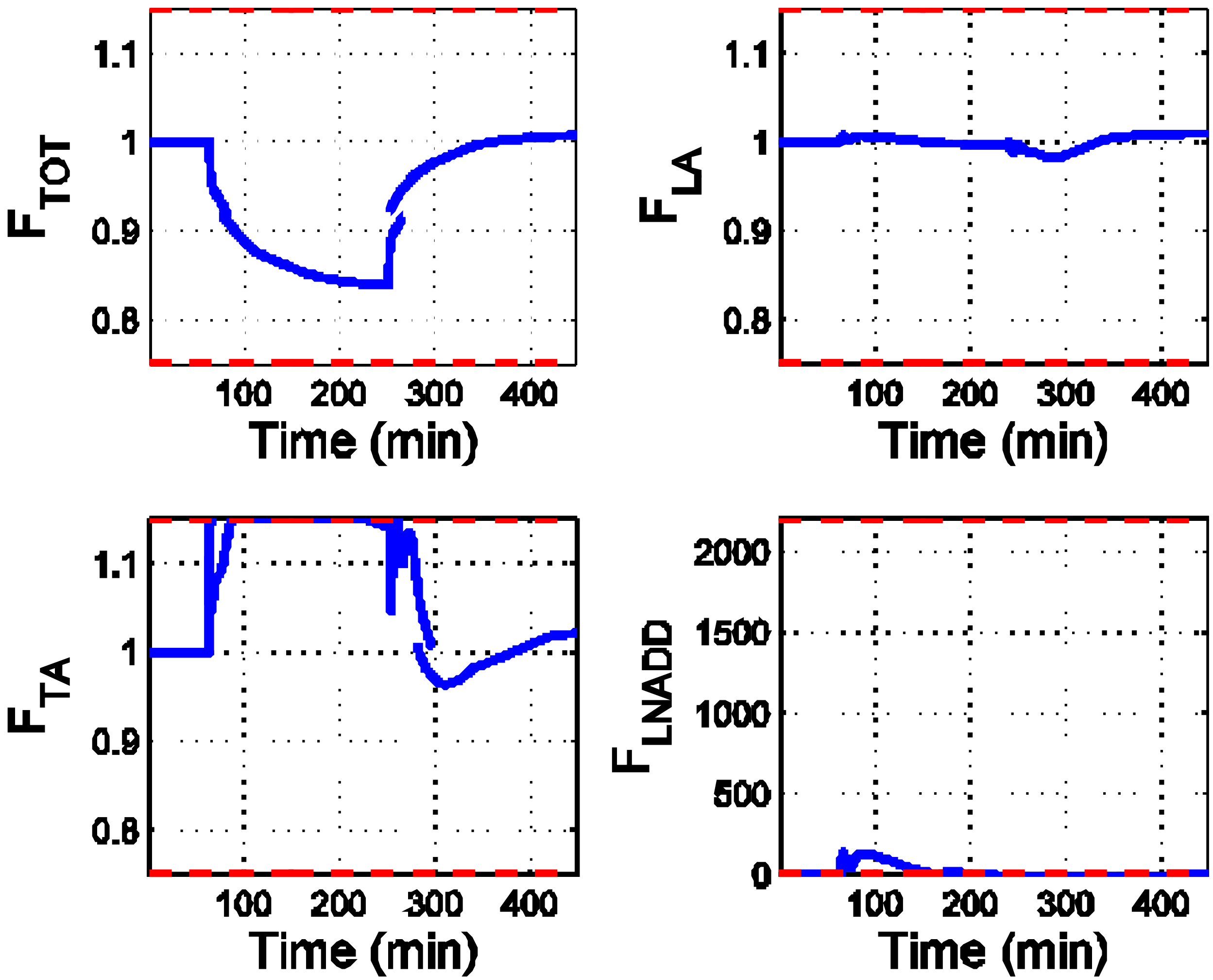
LMPC Formulation for Upper Column

- Controller variables (2)
  - Log transformed N_2 waste composition
  - Log transformed O_2 product purity
- Manipulated variables (5)
  - Feed flowrates of total air, liquid air & turbine air
  - Liquid N_2 addition rate to top of column
  - Gaseous O_2 production rate
- Constraints
  - Linear step response model identified from step tests on Aspen model
  - Manipulated variable bounds
  - Composition bounds
- Tuning
  - Sampling time = 1 min
  - Prediction horizon = 4 hr
  - Control horizon = 30 min
  - Weighting matrices chosen by trial-and-error

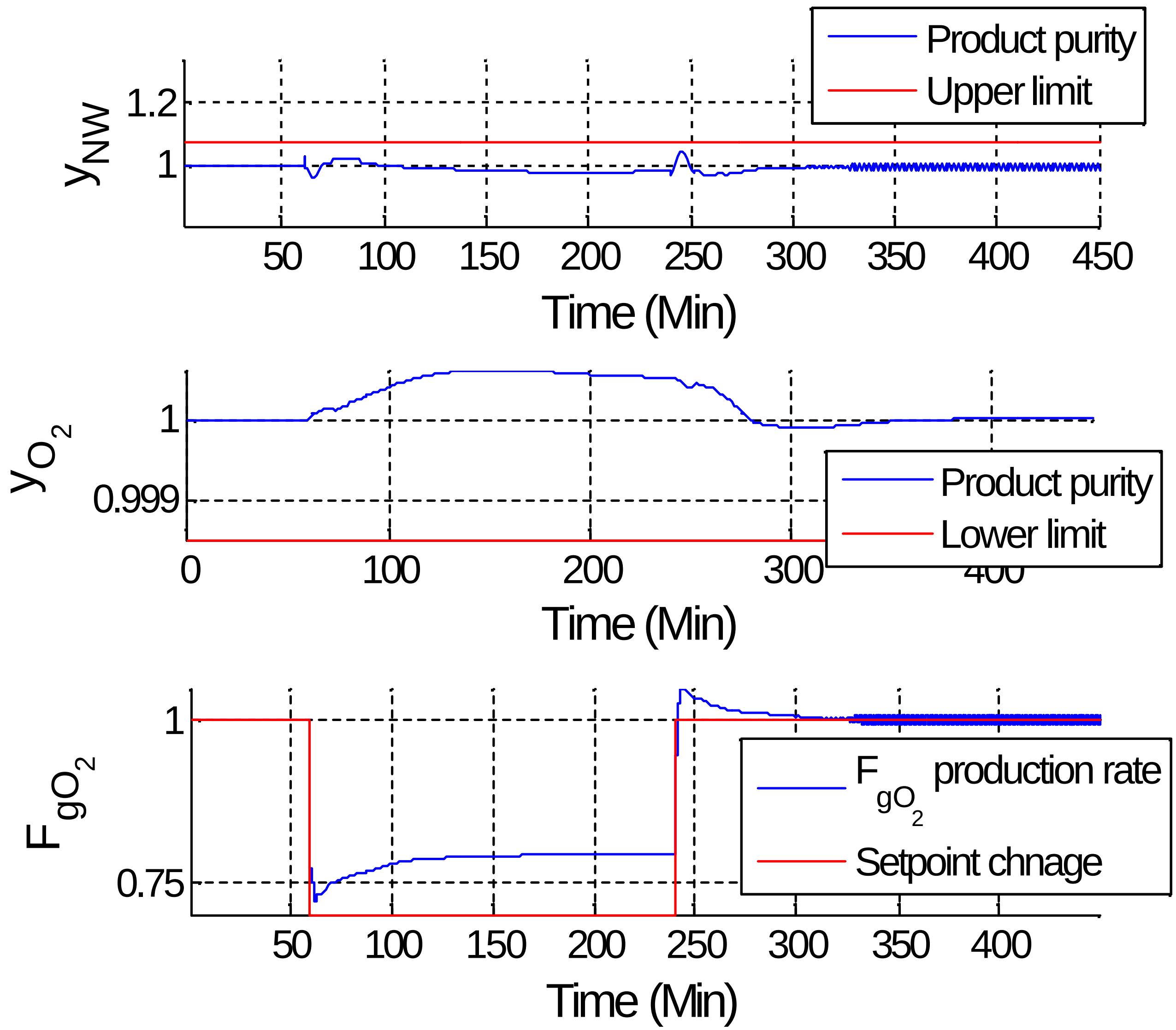
15% Production Rate Changes



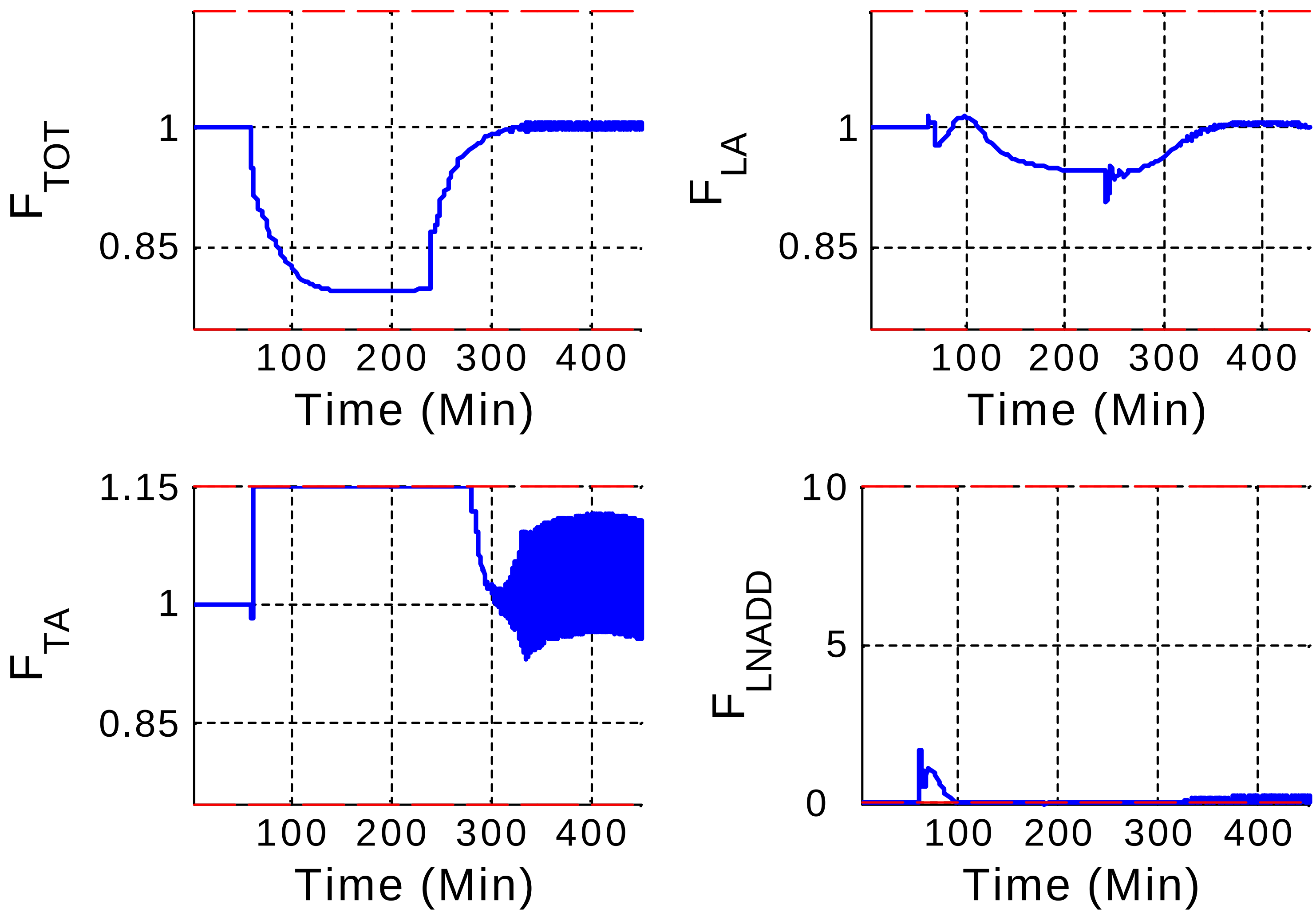
15% Production Rate Changes



30% Production Rate Changes

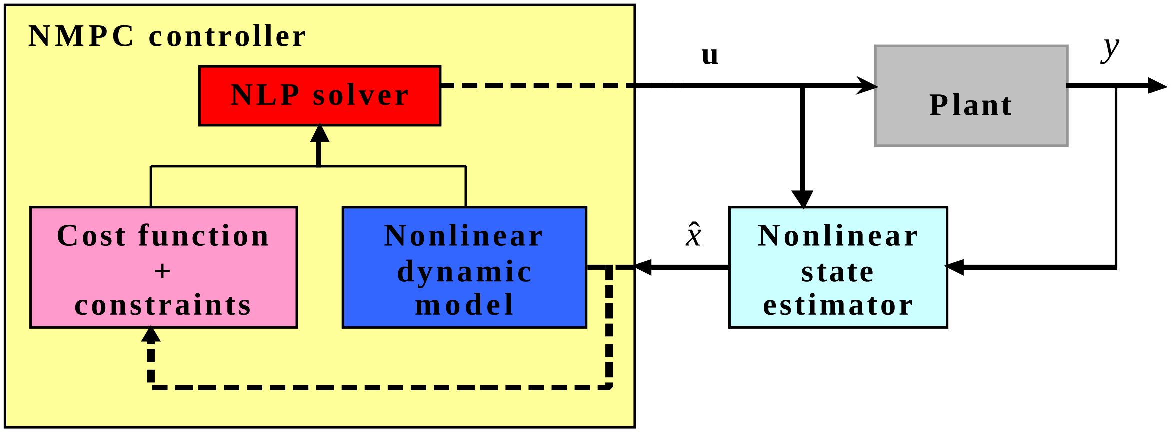


30% Production Rate Changes



Summary – Part 1

- Linear model predictive control (LMPC) is the industry standard for controlling constrained multivariable processes
- LMPC performance depends strongly on the accuracy of the linear dynamic model
- LMPC can perform very poorly for highly nonlinear processes or moderately nonlinear processes that operate over wide regions
- An extension of LMPC based on nonlinear controller design models is needed for such processes

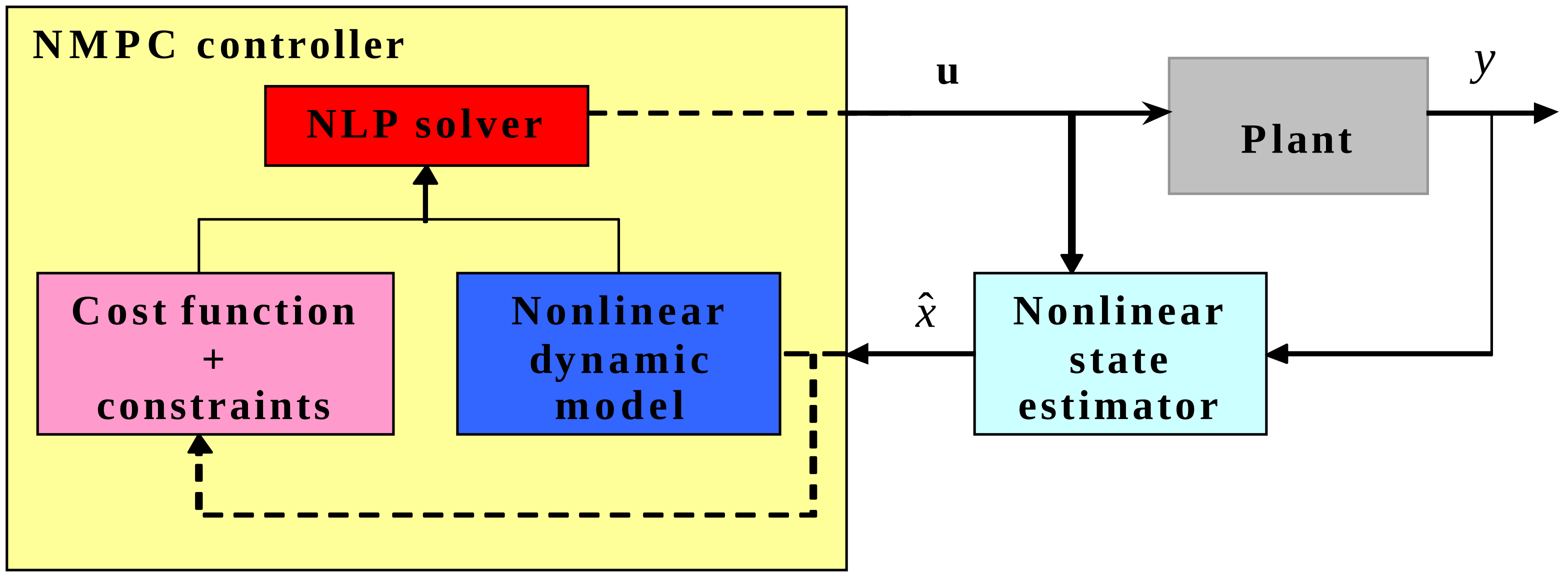


2. Introduction to Nonlinear Model Predictive Control

Desirable MPC Features

- Multivariable compensation
  - No pairing of input & output variables required
- Constraint handling capability
  - Input & output constraints explicitly include in controller calculation
- Model flexibility
  - A wide variety of linear dynamic models can be accommodated
- Receding horizon formulation
  - Allows updating of model predictions with measurement feedback
- On-line implementation
  - Simple & robust quadratic program
- Would like to retain these features in nonlinear extension

Nonlinear Model Predictive Control



- Nonlinear model
  - Better prediction accuracy than linear model
  - Much more difficult to obtain
- Nonlinear program (NLP)
  - Necessitated by nonlinear model
  - More difficult to implement than LMPC
- Nonlinear state estimator
  - Necessary to generate unmeasured state variable

Nonlinear Process Modeling

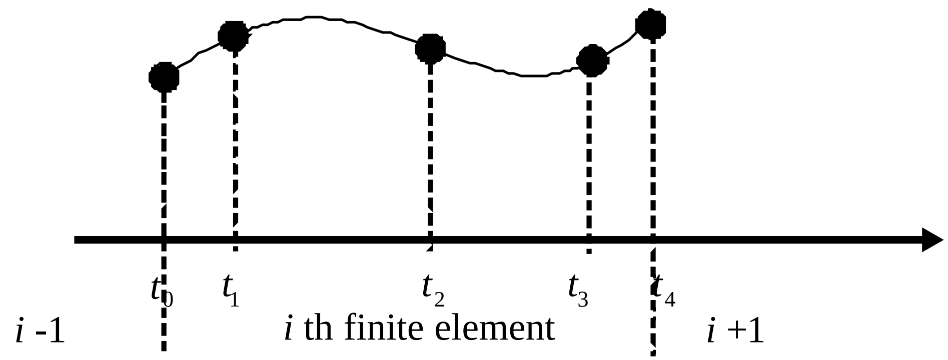
- First-principle models
  - Requires understanding of process fundamentals
  - Derived from conservation principles
  - Parameters obtained from literature & estimation
  - Most common approach for NMPC
  - Profit NLC (Honeywell)
- Empirical models
  - Artificial neural networks, NARMAX models, etc.
  - Highly overparameterized & data intensive
  - Poor extrapolation capabilities
  - Suitability for NMPC being demonstrated
  - Apollo (Aspen Technology)
- Development of accurate, computationally efficient nonlinear models remain a major obstacle to NMPC

NMPC Solution Techniques

- Sequential solution
  - Iterate between NMPC optimization & model solution codes
  - Inefficient & non-robust for large problems
- Simultaneous solution
  - All discretized model variables posed as decision variables
  - Produces large-scale NLP problems
  - Routinely applied to low-dimensional process models
  - Moderate size problems solvable with commercial codes
  - Limited by problem size
- Multiple shooting
  - Hybrid of the sequential & simultaneous methods
  - Promising method under development

Model Discretization

- Fundamental model
  - Nonlinear differential-algebraic equations (DAEs)
  - Must be posed as algebraic constraints for NLP solution
  - Requires discretization in time
  - Many methods available
- Orthogonal collocation
  - Highly accurate discretization method
  - Model equations approximated at fixed collocation points
  - Difficult to approximate sharp solutions
  - Produces dense Jacobian matrix



- Finite elements
  - Convenient method for NMPC
  - Divide prediction horizon into N finite elements
  - Place n collocation points in each finite element
  - Accurate & efficient

Simultaneous NMPC Formulation

- Basic elements
  - Quadratic objective function
  - Bounds on input & output variables
  - Nonlinear algebraic equation constraints arising from model discretization
  - Yield a nonlinear programming (NLP) problem
- NLP characteristics
  - Many decision variables & constraints
  - Computationally difficult
  - Non-convex à existence of local minima
  - Sensitive to equation & variable scaling
  - Convergence not guaranteed

$$
\begin{aligned}
\text{Min} \quad & f(X) \\
\text{St.} \quad & g(X) \le 0 \\
& h(X) = 0 \\
& X^L \le X \le X^U
\end{aligned}
$$

Representative NMPC Products

Profit NLC

- Developed jointly by Honeywell and PAS Inc. & marketed by Honeywell
- Based on fundamental nonlinear models
- State estimation strategy not described
- Most reported applications to polymer processes
- Basell, British Petroleum, Chevron Phillips, Dow

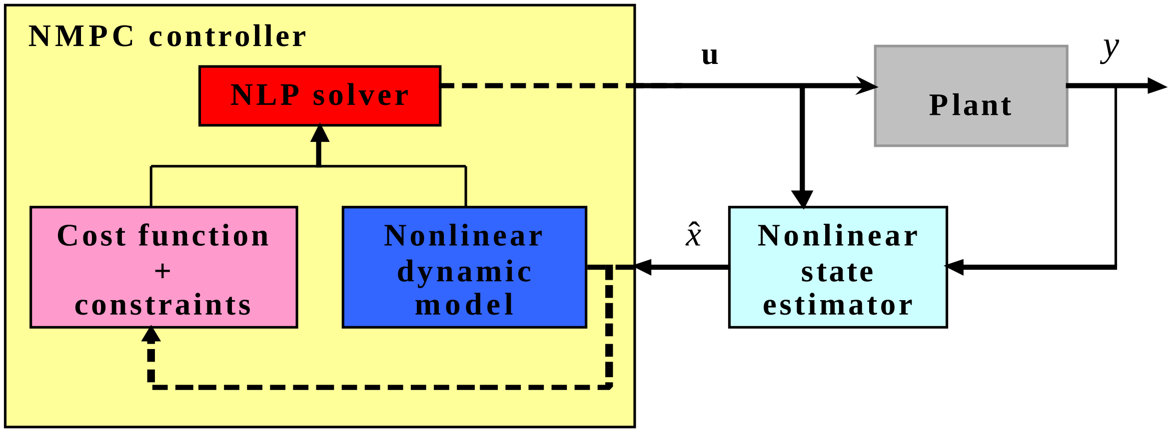
Apollo

- Developed & marketed by Aspen Technology
- Based on empirical nonlinear models of gain-time constant-delay form
- State estimation performed with extended Kalman filter
- Designed for polymer processes
- Industrial applications underway

Process Perfecter from Pavilon Technologies & Rockwell Automation

Summary – Part 2

- Nonlinear model predictive control (LMPC) is extension of LPMC based on nonlinear dynamic models & optimization
- NMPC offers the potential for improved performance when applied to highly nonlinear processes
- The most widely accepted NMPC approach is based on fundamental model discretization & simultaneous solution
- Commercial NMPC products are available & have been successfully applied to polymer processes
- A major challenge to successful NMPC application to other processes is real-time implementation



3. Real-Time Implementation Issues

NMPC Problem Size

- Simultaneous solution approach
  - Every state variable at every discretization point is treated as a decision variable
  - Typically produces a large NLP problem
- Problem size determined by:
  - Order of the original dynamic model
  - Number of discretization points
  - Prediction & control horizons
- On-line implementation
  - Requires repeated solution of NLP problem at each sampling interval
  - Real-time implementation can be non-trivial

Real-Time Implementation

- NMPC requires on-line solution of a large, non-convex NLP problem at each sampling interval
  - Typical industrial sampling intervals ~1 minute
  - Controller must reliably converge within the sampling interval
- Potential problems
  - Controller converges to a poor local minimum
  - Controller fails to converge within the sampling interval
  - Controller diverges
- Real-time implementation techniques that mitigate these problems are essential
  - Not a focus of typical academic studies
  - Not openly reported by NMPC vendors & practitioners

Controller Design Model

- Simultaneous solution method
  - Well suited for processes that can be described by nonlinear models of moderate order (<50 DAEs)
  - Yield reasonably sized NMPC problems (~10,000 decision variables)
  - Directly applicable to most polymer reactor models
  - Distillation column models are problematic due to their high order
- Nonlinear model order reduction
  - Reduce model order while retaining the essential dynamical behavior
  - Few generally applicable methods are available
  - Single perturbation analysis, proper orthogonal decomposition
  - Typically method must be customized to specific process

Model Discretization

- Orthogonal collocation on finite elements
  - Divide prediction horizon into N finite elements with each finite element corresponding to a sampling interval
  - Place n collocation points in each finite element
  - Typically use large N (~100) and small n (<5)
- Prediction horizon
  - Chosen according to the steady-state response time
  - Dynamics change slowly near end of prediction horizon where the inputs are held constant
- Finite elements of non-equal length
  - Use regularly spaced elements over control horizon
  - Use increasingly wider spaced elements after the control horizon
  - Reduces number of discretization points
  - Implementation problem dependent

NLP Solution Code

- Wide variety of NLP codes are available
  - Successive quadratic programming (NPSOL)
  - Generalized reduced gradient methods (CONOPT)
  - Interior point methods (IPOPT)
- Problem dependence
  - Particular codes work better for specific problems
  - General guidelines available but successful implementation requires match of NLP problem and code
  - Typically must be determined by trial-and-error experimentation & code tuning
  - Facilitated by general purpose optimization modeling tools such as AMPL and GAMS
  - Problem/code matching reduces the number of iterations and/or the time per iteration

Derivative Information

- First-order and second-order derivatives
  - Required for discretized model & constraint equations with respect to the decision variables
  - The Jacobian & Hessian matrices tend to be large & ill-conditioned
  - Can be numerically calculated by finite difference
  - Very time consuming & subject to numerical errors
- Analytical derivative calculation
  - Derivative exactly calculated from analytical formulas
  - Facilitated by automatic differentiation capabilities of optimization modeling languages (AMPL)
  - Improves NLP code efficiency & robustness

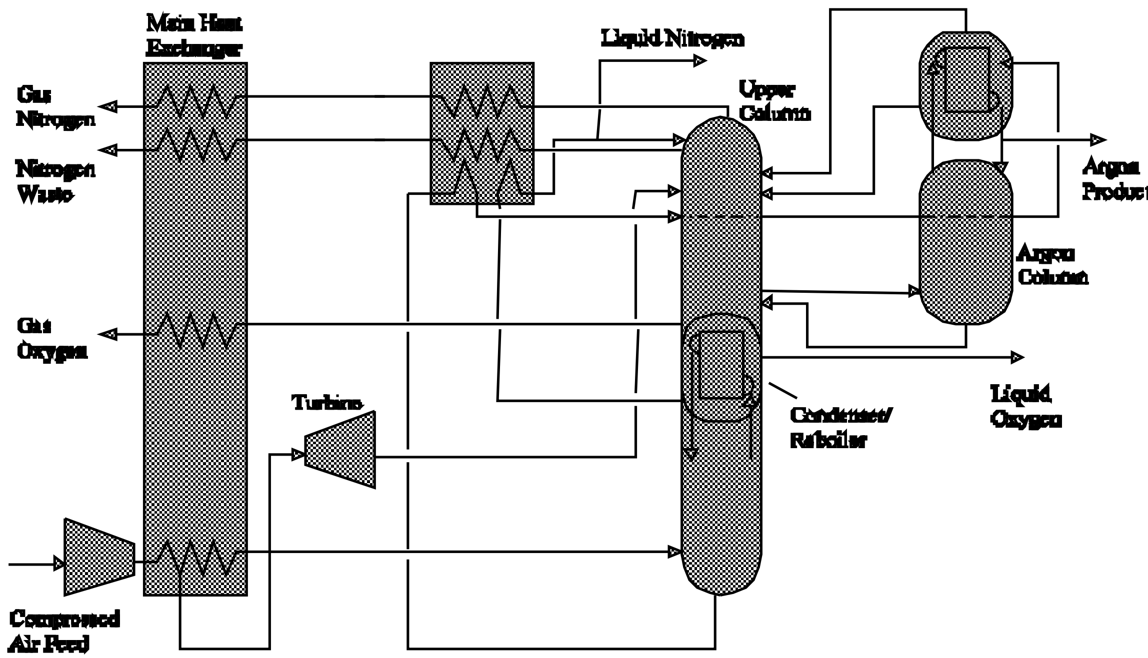
Controller Initialization

- NLP solution
  - An initial guess of the solution X^0_k is required at each sampling interval k
  - Convergence properties depend strongly on X^0_k
  - Need to generate a good X^0_k near the optimal solution
- Warm start strategy
  - Use converged solution from previous iteration X_{k-1} to generate X^0_k
  - Set $X^0_{k+j|k} = X_{k+j|k-1}$ and $X_{k+p|k} = X_{k+p-1|k-1}$
  - Reduces the number of NLP iterations
- Caveats
  - Not guaranteed to produce fast convergence
  - Not effective immediately following setpoint or disturbance change

$$\begin{aligned} \text{Min} \quad & f(X) \\ \text{St.} \quad & g(X) \le 0 \\ & h(X) = 0 \\ & X^L \le X \le X^U \end{aligned}$$

Summary – Part 3

- The NMPC simultaneous solution method yields large & non-convex NLPs
- The NLP must be solved efficiently & robustly during each sampling interval
- Modifications of the basic NLP strategy are needed to facilitate real-time implementation
  - Nonlinear model order reduction
  - Customized model discretization strategies
  - Matching of discretized model with NLP code
  - Analytical derivative calculation
  - Warm start strategies



4. Nonlinear Control of an Air Separation Column

Fundamental Model of Upper Column

- Assumptions
  - Similar to those used for Aspen model development
  - Also assume negligible vapor phase holdups & linear pressure drop across column
- Equations
  - Dynamic mass & component balances
  - Steady-state energy balances
  - Non-ideal vapor-liquid equilibrium different from Aspen model
  - Reboiler level & overhead pressure controllers
  - Lower column effect on upper column described by empirical linear models
- Dimensionality
  - 180 differential equations & 137 algebraic variables
  - About 1900 intermediate variables for thermodynamic model

Standard NMPC Formulation

- Problem characteristics
  - 2 output & 5 input variables
  - 317 state variables & 1900 thermodynamic variables
  - Discretization produces ~500,000 decision variables
  - Very challenging NLP problem that pushes the state-of-the-art
  - Application of simultaneous solution method have been limited to open-loop dynamic optimization
- Possible solutions
  - Develop customized solution techniques that exploit problem structure
  - Develop real-time implementation strategies

Real-Time Implementation

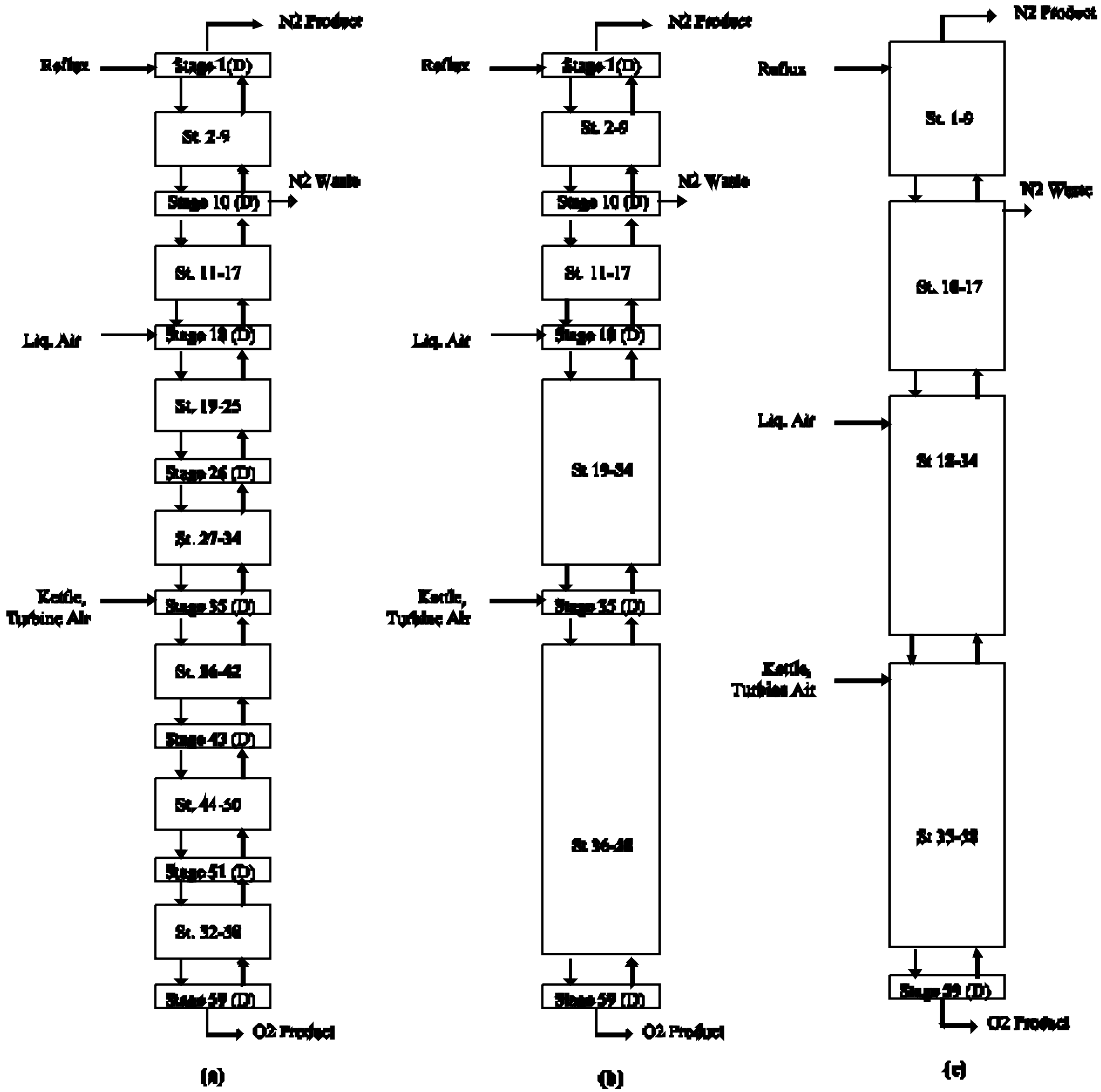
- Use compartmental model
  - Jacobian & Hessian matrices become more sparse
  - Significantly reduces the time for each NLP iteration
- Allow finite elements to have non-uniform lengths
- Optimize the NLP solver options
  - Requires code expertise
  - Significantly reduces the number of NLP iterations
- Use warm start strategy
- Calculate derivative information analytically
- Less important strategies
  - Scale the variables & constraint equations
  - Implement setpoint changes as ramps or exponentials

Compartmental Modeling

- Fundamental idea
  - Divide column into several sections (compartments)
  - Only describe the overall (slow) dynamics of each compartment with a differential equation
  - Describe the individual stage (fast) dynamics with algebraic equations
- Compartmental model characteristics
  - Provides perfect steady-state agreement with fundamental model
  - Fewer differential equations but more algebraic equations
  - Highly accurate if a sufficient number of compartments is used
- Advantages for NMPC
  - Compartmentalization yields a more sparse model structure that can be exploited by NLP codes

S. Khowinij et al., Separation and Purification Technology, 46, 95-109, 2005

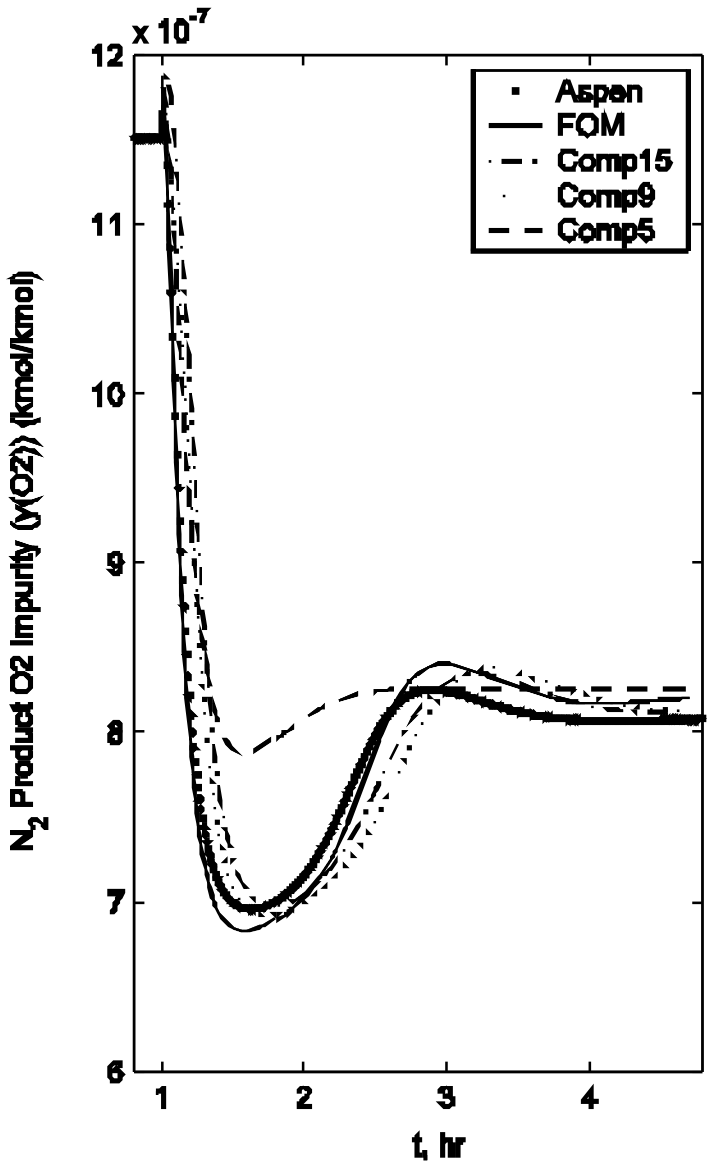
Upper Column Compartmentalization

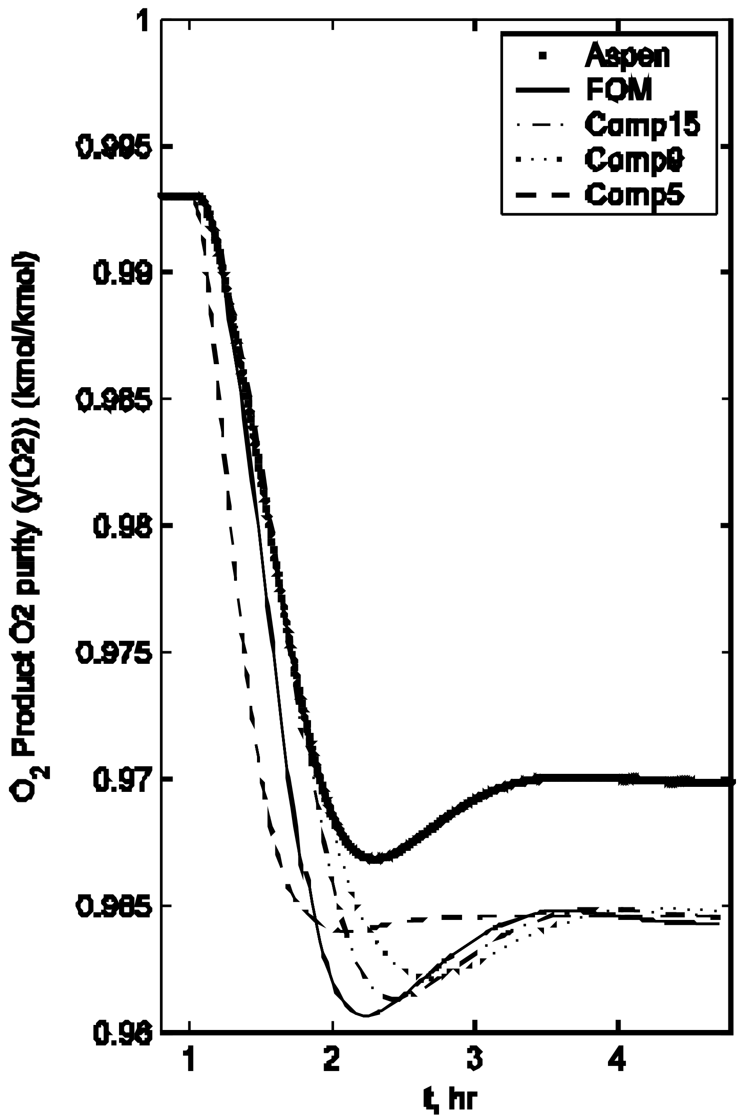


Model	ODEs	AEs
Fundamental	180	137
15 compartment	48	269
9 compartment	30	287
5 compartment	18	299

S. Bian et al., Computers & Chemical Engineering, 29, 2096-2109, 2005.

Comparison of Compartmental Models

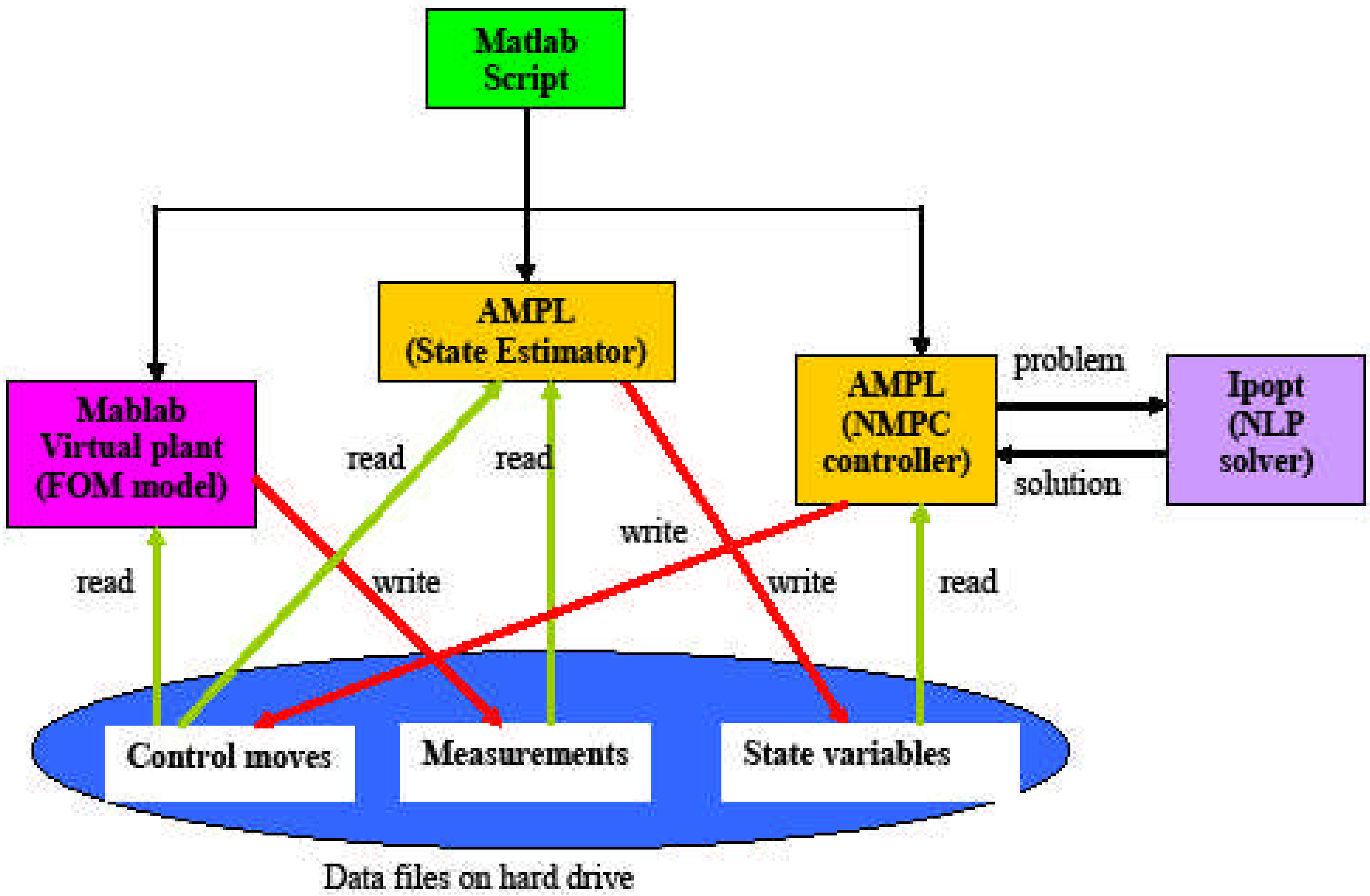




NMPC Formulation for Upper Column

- Largely unchanged from LMPC formulation
  - Sampling time = 2 min
  - Prediction horizon = 4.3 hr
  - Control horizon = 20 min
- Discretized dynamic model equations
  - Compartmental model
  - Nonlinear equality constraints
- NLP solution
  - Interior point code IPOPT within AMPL
  - Less success with popular solver CONOPT
  - AMPL coupled to fundamental model in MATLAB

NMPC Formulation for Upper Column



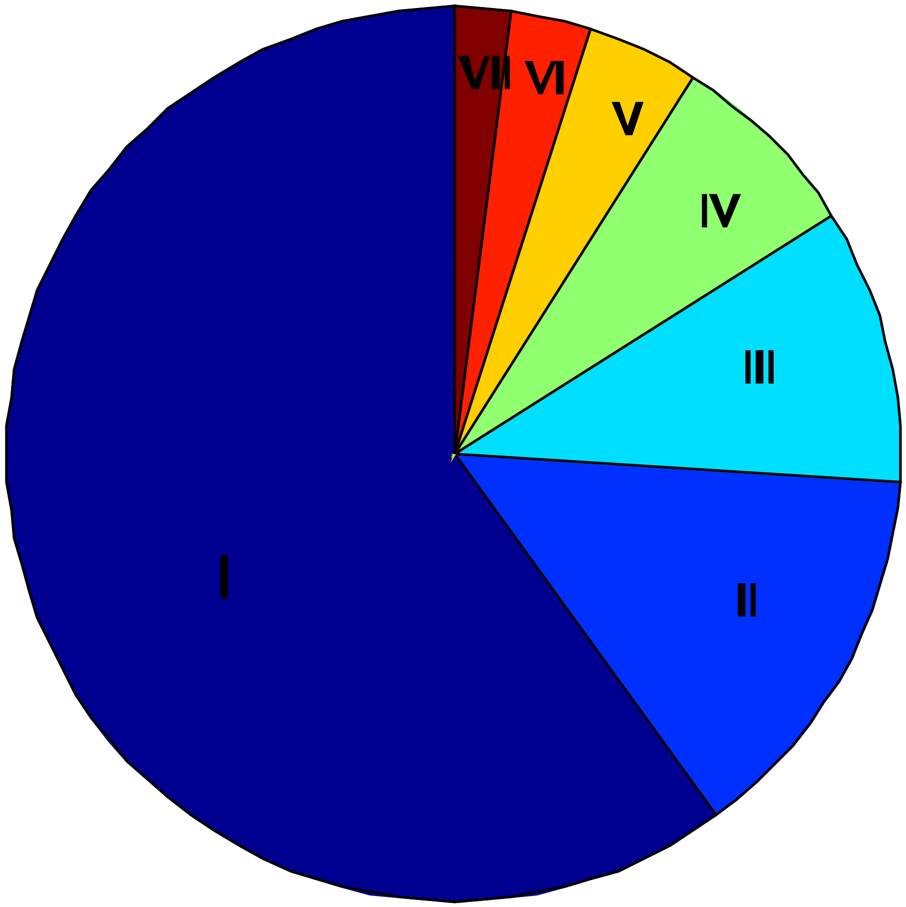
Reducing Real-Time Computation

Initial CPU time ~40 min/iteration

I. Use finite elements with non-uniform lengths
II. Use compartment model
III. Optimize the NLP solver options
IV. Analytically calculate the Jacobian & Hessian matrices
V. Scale the variables & constraints
VI. Use warm start strategy
VII. Implement setpoint changes as ramps

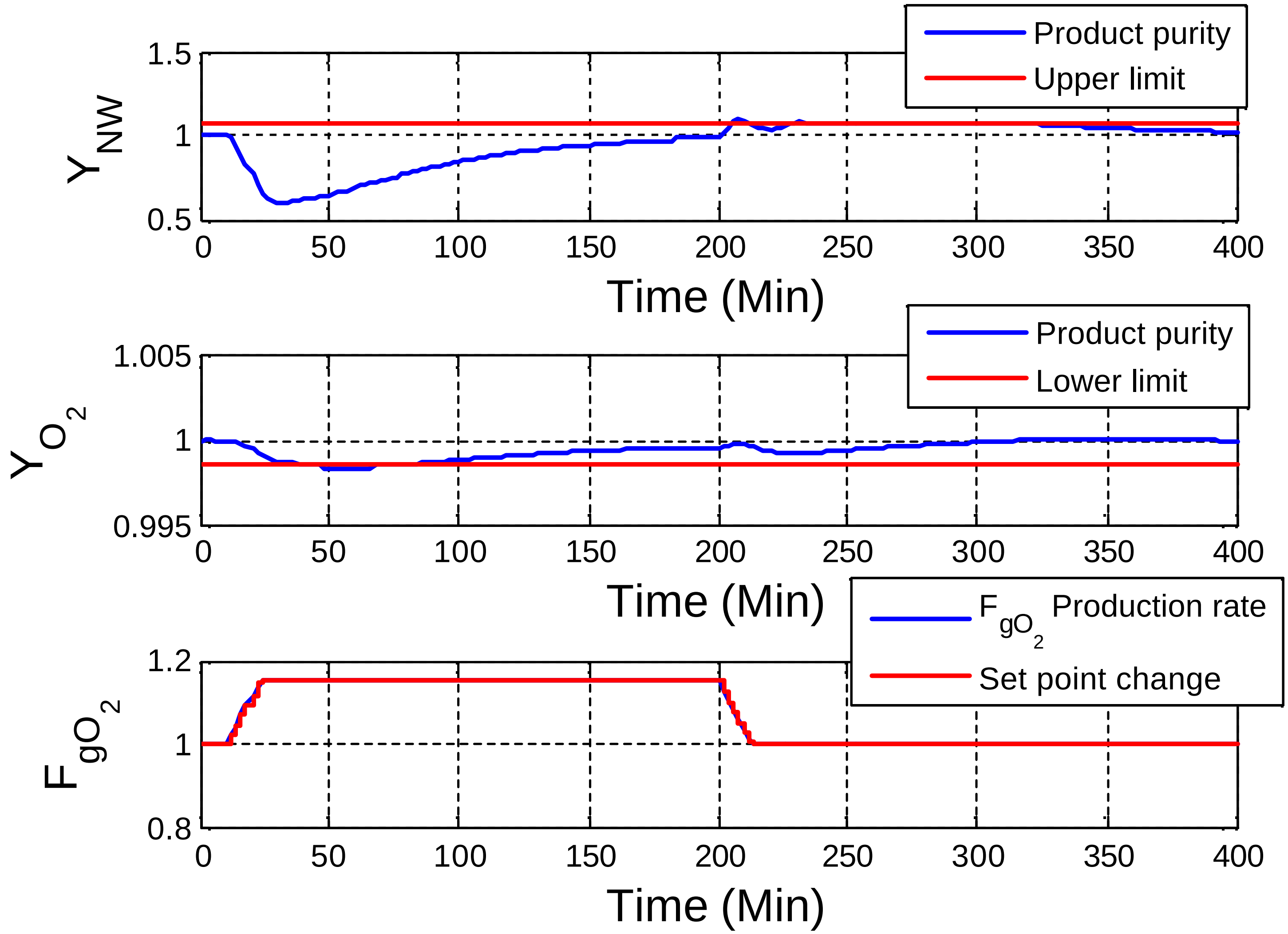
Final CPU time ~2 min/iteration

12780 decision variables & 12730 constraints

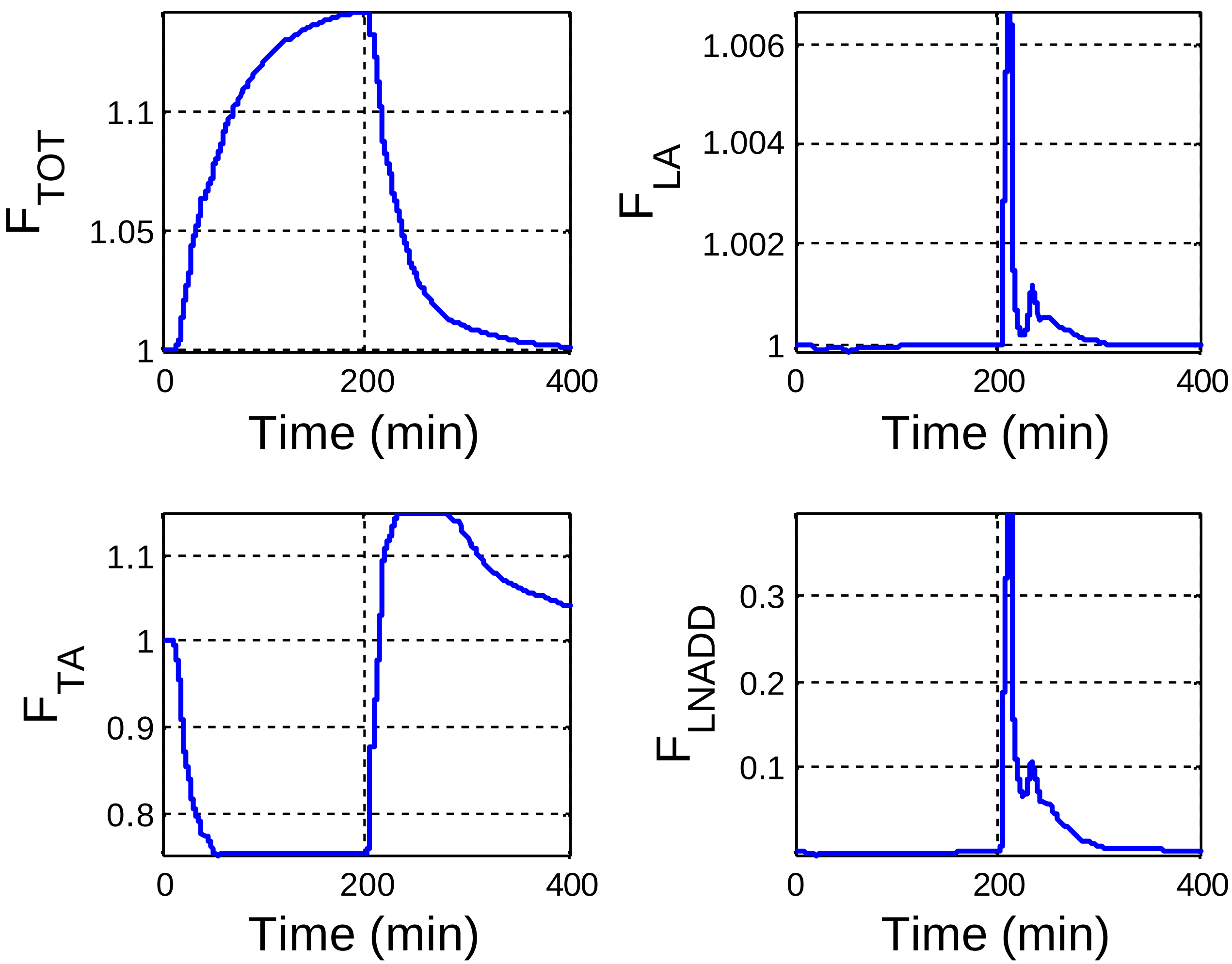


Contribution of each strategy to reduction in worst case NMPC CPU time

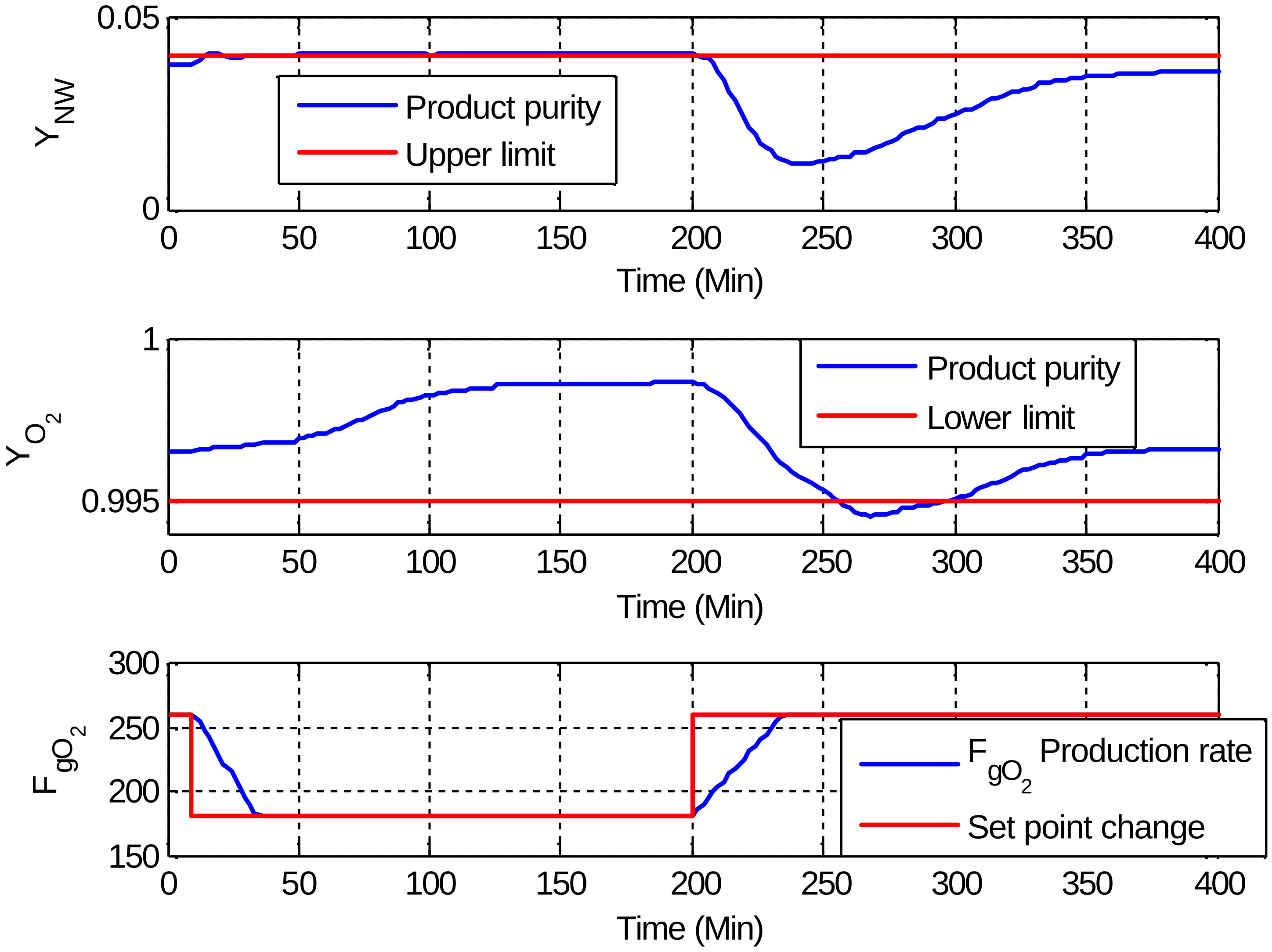
15% Production Increase



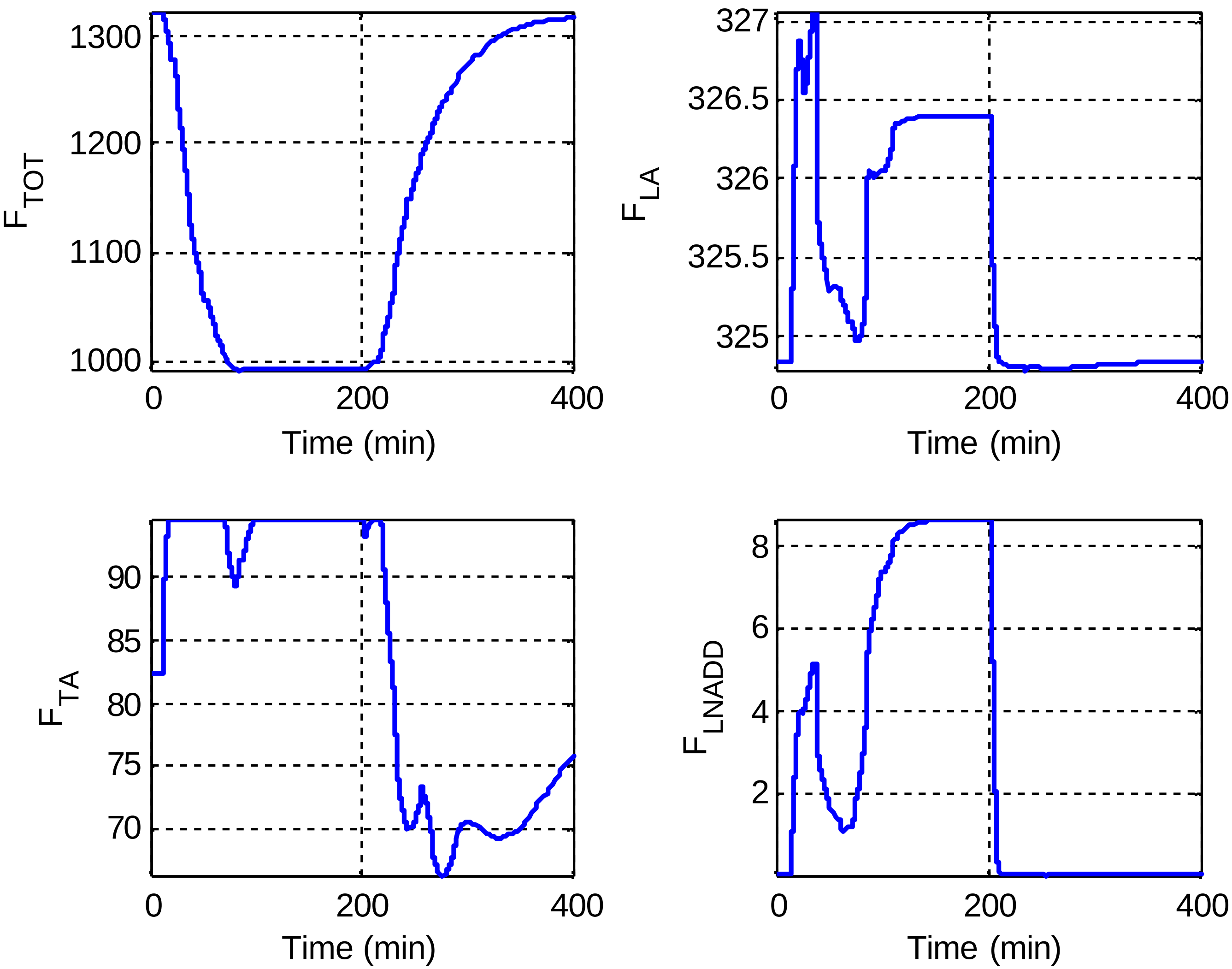
15% Production Increase



30% Production Decrease

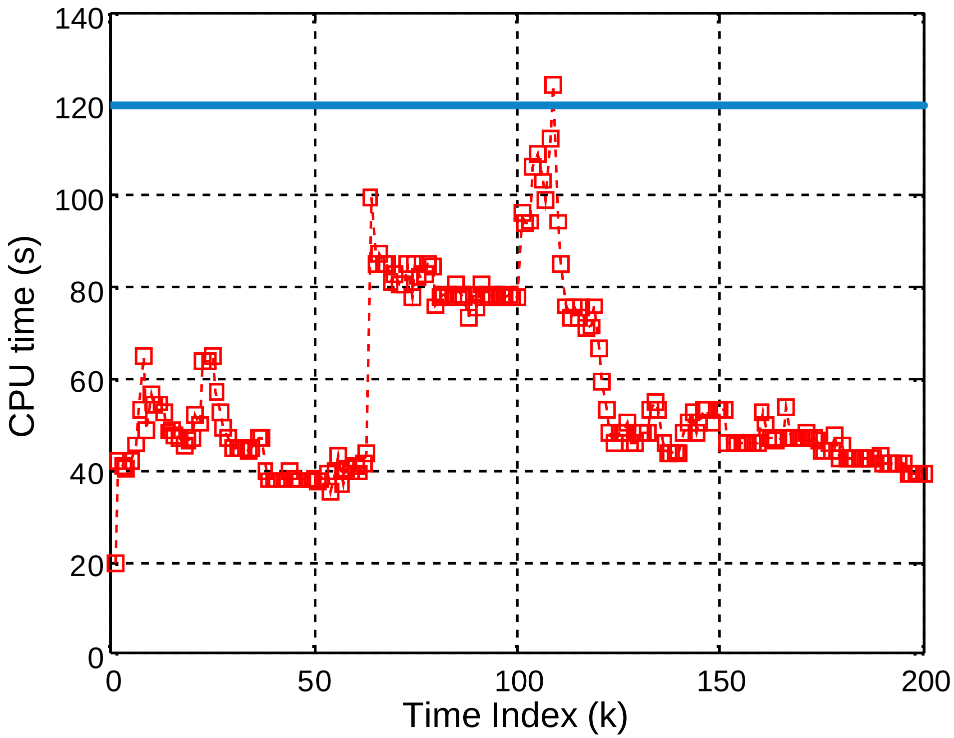


30% Production Decrease

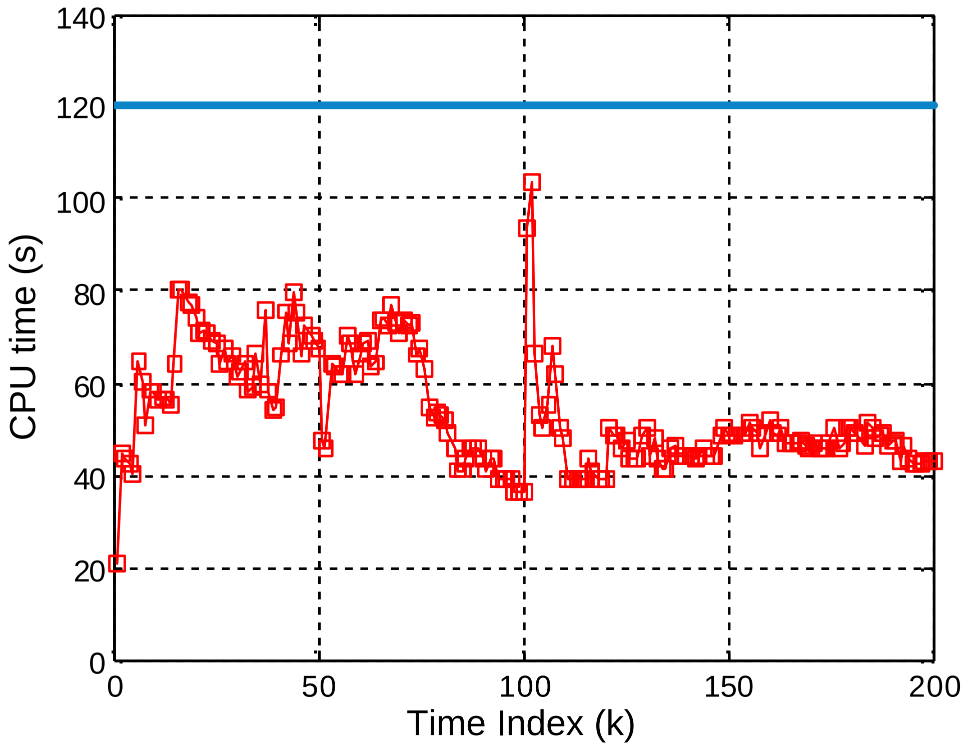


NMPC CPU Times per Iteration

15% Production Increase



30% Production Decrease



Summary – Part 4

- Direct application of NMPC to the air separation column yielded a very large NLP problem not suitable for real-time implementation
- The combination of reduced-order modeling and several real-time implementation strategies reduced computation time by 2000%
- NMPC provided good performance for large production rate changes that proved problematic for LMPC
- NMPC development required considerable time and effort

Final Comments

- NMPC is a promising technology for nonlinear plants subject to large dynamic changes
- Availability of an accurate nonlinear model is paramount
- Real-time implementation strategies are often necessary for reducing computation
- Nonlinear receding horizon estimation is a promising approach for generating estimates of unmeasured state variables (not shown here)
- The time and effort required for NMPC development and maintenance must be justified