



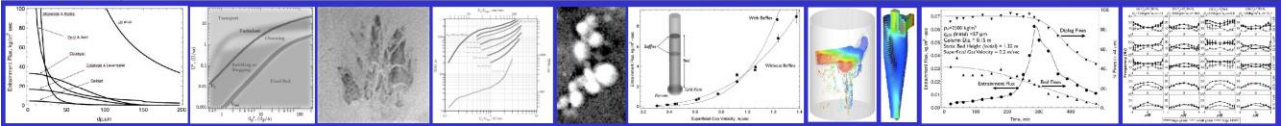
# Review of Fluid-Particle Drag Experimental Validation

Casey LaMarche, Ben Freireich, Ray Cocco



Particulate Solid Research, Inc.

**PSRI** *Applying the Fundamentals*  
Particulate Solid Research, Inc.



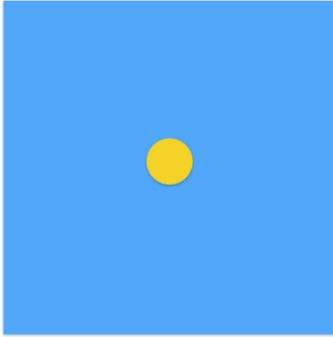
## Outline

- Problem of interest
- Methods of approach
- Verification and Validations
- Summary

What's the question?  
 How has it been answered?  
 Are the answers any good?  
 So what?

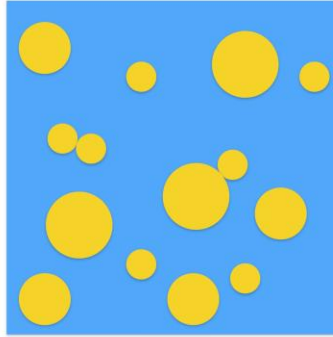
# Spectrum of Interactions

Single Particle



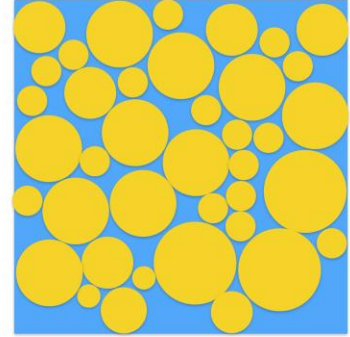
Drag Coefficient

Intermediate?



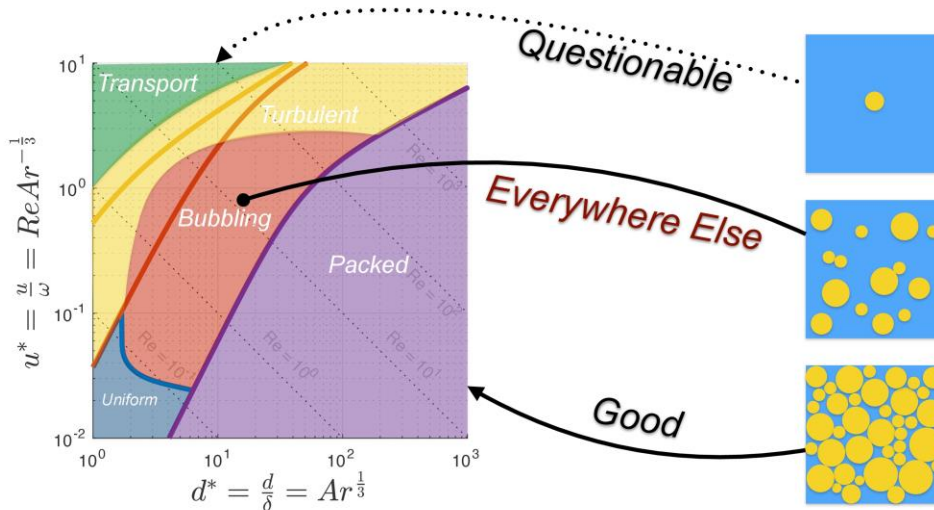
How to transition?

Packed Bed



Pressure Drop

## Where Do We Care?



# Analytical Single Particle Drag

## Stokes/Creeping Flow

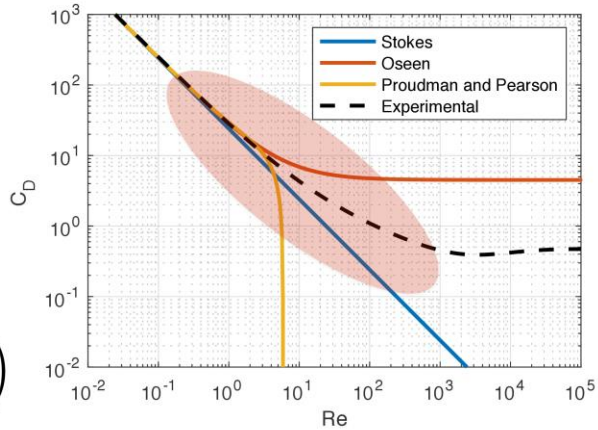
$$C_{\text{Stokes}} = \frac{24}{Re}$$

## Perturbed

$$C_{\text{Oseen}} = \frac{24}{Re} \left( 1 + \frac{3}{16} Re \right)$$

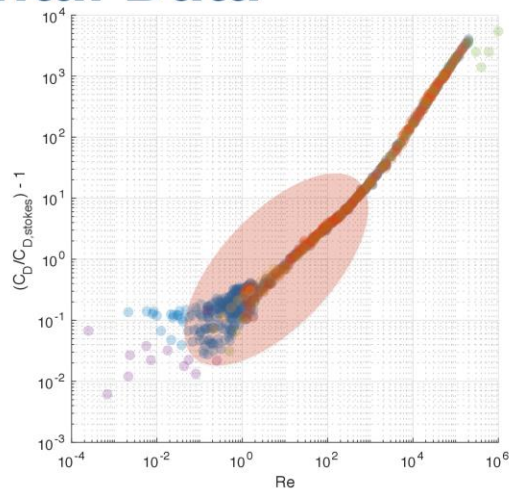
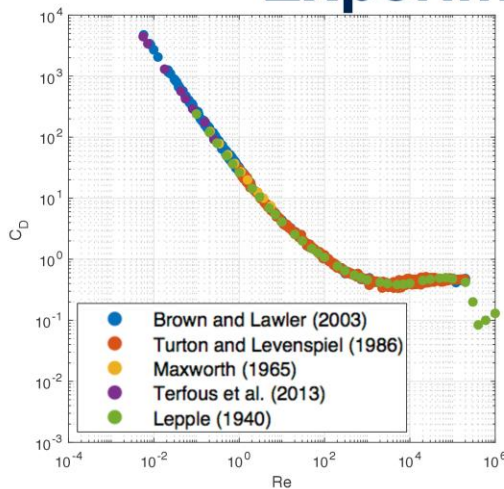
## Perturbed<sup>2</sup>

$$C_{\text{P\&P}} = \frac{24}{Re} \left( 1 + \frac{3}{16} Re - \frac{9}{160} Re^2 \ln Re \right)$$



*Accurate only for  $Re < 1$*

# Experimental Data



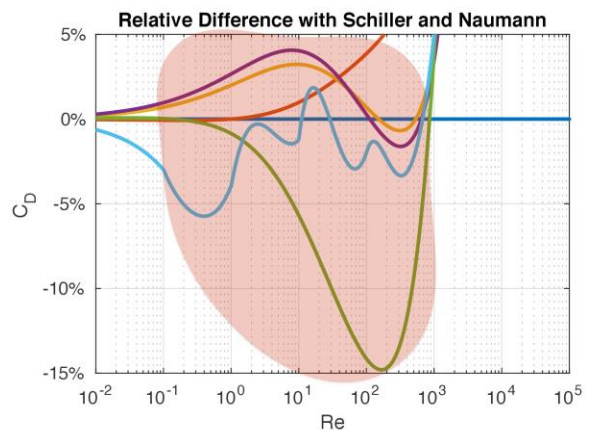
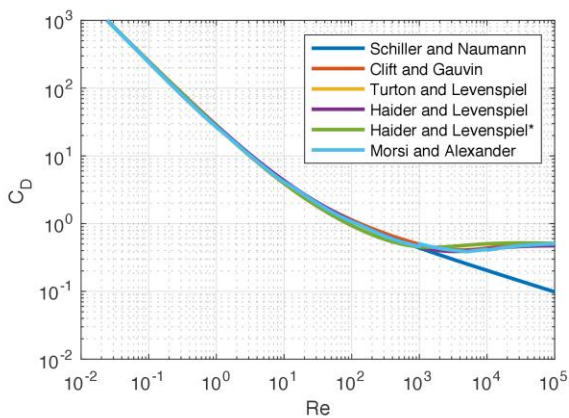
*Lots of scatter for  $Re < 10$*

# Experimental Correlations

$$C_D = \frac{24}{Re} (1 + ARe^B) + \frac{C}{1 + DRe^{-E}}$$

	A	B	C	D	E
Schiller and Naumann	0.1500	0.6870	0		
Clift and Gauvin	0.1500	0.6970	0.4200	42500.00	1.16
Turton and Levenspiel	0.1730	0.6570	0.4130	16300.00	1.09
Haider and Levenspiel	0.1806	0.6459	0.4251	6880.95	1
Haider and Levenspiel*	0.1402	0.6530	0.4610	2682.50	1

# Experimental Correlations



*Is single particle known?*

# Experimental Correlations

$$C_D = \frac{A}{Re} \frac{1 - \varepsilon}{\varepsilon^2} + \frac{B}{\varepsilon^2}$$

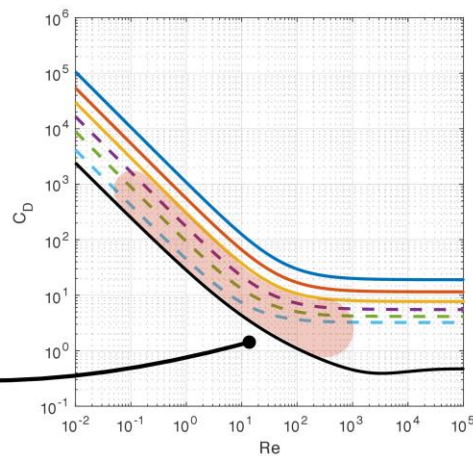
	A	B
Darcy	$1.33d^2k^{-1}$	0
Carman-Kozeny	240	0
Ergun	200	2.33
MacDonald et al.	240	2.40
MacDonald et al.*	240	5.28

S.A. Morsi and A.J. Alexander, J. Fluid Mech., 55 (1972) 193

## Typical Shape

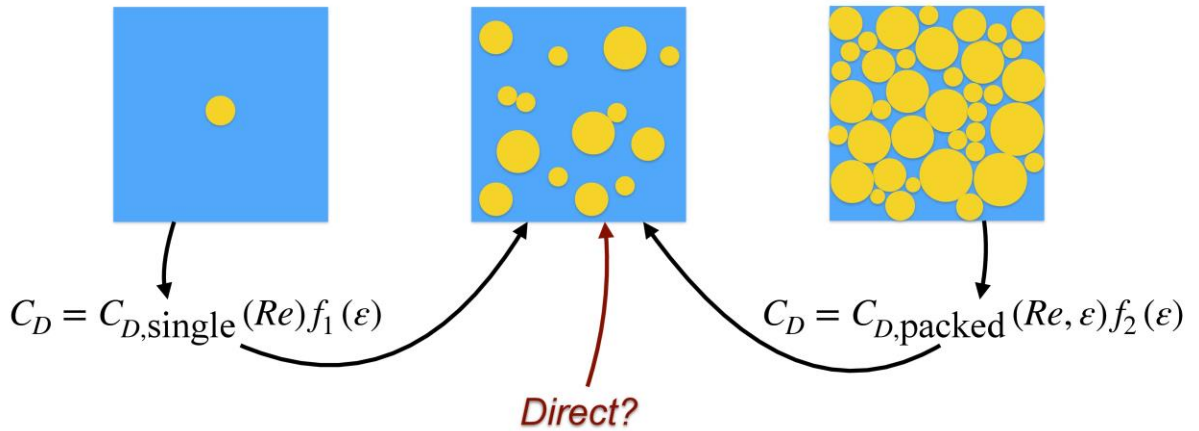
*Ergun for  $0.10 < \varepsilon < 0.44$*

*Haider and Levenspiel*

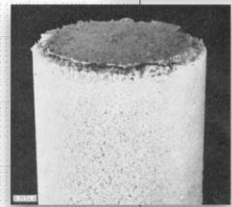
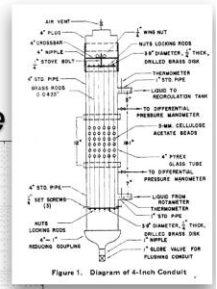
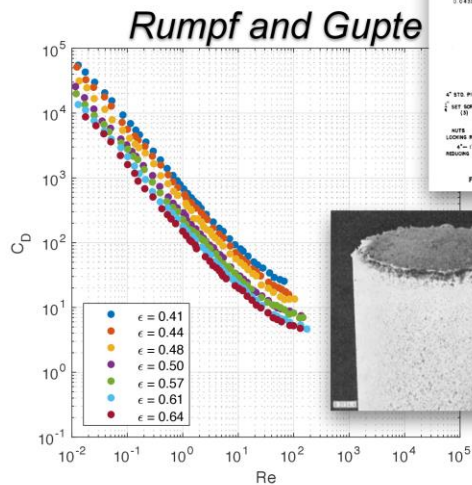
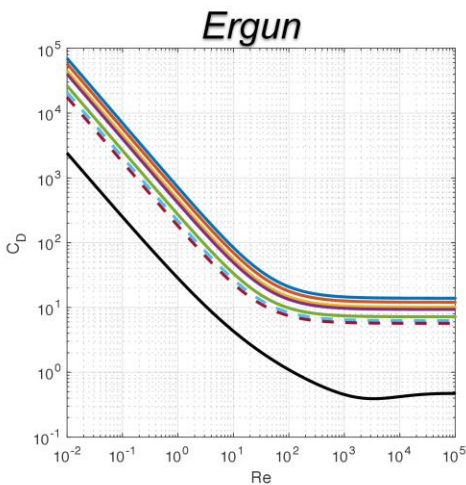


*Meaningful only for  $\varepsilon > 0.4$*

# What About the Middle?

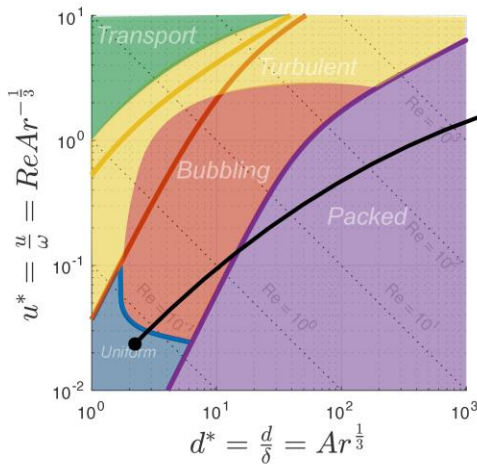


# Fixed Array Experiments



Happel Epstein (1954)  
Rumpf and Gupte (1971)

# Bed Expansion Experiments



- Uniformly Fluidize

Only Here

Severe  
Limitation

- Force Balance

$$F_D = F_g - F_b$$

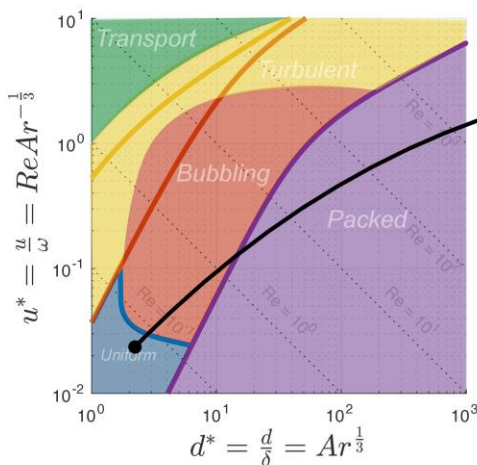
- Assume Voidage Multiplier

$$F_D = F_{D,\text{single}} f(\varepsilon) \quad \text{Must Assume } C_{D,\text{single}}$$

- Experimentally Determine

$$f(\varepsilon) = \frac{4}{3} \frac{Ar}{Re} C_{D,\text{single}}^{-1} \approx \varepsilon^{-n}$$

# Hindered Settling Experiments



- Uniformly Fluidize

Only Here

Severe  
Limitation

- $U = 0$  then watch surface drop

$$U_T = U_{\text{surface}}$$

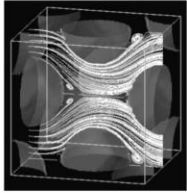
- Assume Voidage Multiplier

$$U_T = U_{T,\text{single}} f(\varepsilon) \quad \text{Must Assume } U_{T,\text{single}}$$

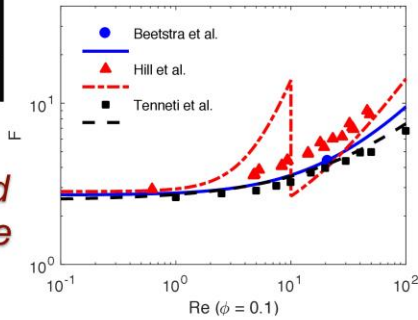
- Experimentally Determine

$$f(\varepsilon) = \frac{U_T}{U_{T,\text{single}}} \approx \varepsilon^{-n}$$

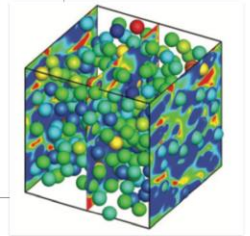
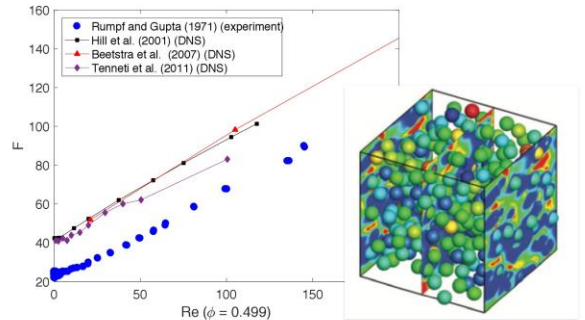
# Direct Numerical Simulation



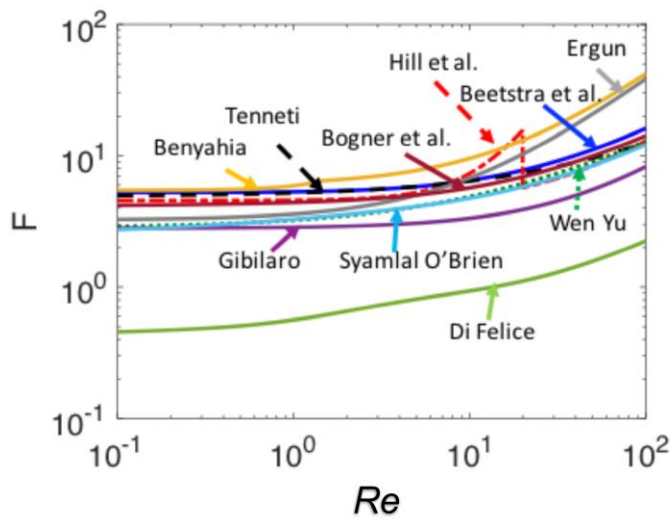
*Very limited domain size*



*DNS over predicts experiments*



# Many Common Methods



*Large variation*



# Summary

- Existing drag measurements are limited to:
  - Single particle (dilute)
  - Packed bed (dense)
  - Uniform fluidization
  - Contrived fixed arrays
- Interpolation requires single particle drag law assumption
- Heterogeneous drag prediction (filtering and EMMS) assumes a homogeneous model

*There is still space  
for work here*

# Acknowledgement

- PSRI Staff
  - Cory Jarvis
  - Bryan Harrison
  - Ryan Foy
- PSRI Membership
  - Clay Sutton
  - Benjamin Amblard
  - Raj Singh

*Questions?*